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GENERALIZATION OF DATA AUGMENTATION TO REDUCE THE NUMBER OF EPOCHS TO AVERAGE

*** Idileisy Torres-Rodríguez [1], Beatriz Peón-Pérez[2], Diamir De Ávila Rodríguez[3], Samuel Cárdenas Herrera[4], Alberto Taboada-Crispí[5]**

[1,5] Universidad Central "Marta Abreu" de Las Villas, Santa Clara, Cuba.
[2] Hospital Manuel Piti Fajardo, Santa Clara, Cuba.
[3, 4] Empresa de Automatización (CEDAI), Santa Clara, Cuba.

[1]http://orcid.org/ 0000-0002-1783-9908, [2]http://orcid.org/ 0000-0003-3632-347X, [3]http://orcid.org/0009-0004-9988-1246, [4]http://orcid.org/0009-0007-3112-198X, [5]http://orcid.org/ 0000-0002-7797-1441

Email:[1] itrodriguez@uclv.edu.cu*, [2]beatrizpeonperez@gmail.com, [3]diamir@cedai.com.cu, [4]samuel@cedai.com.cu, [5] ataboada@uclv.edu.cu

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ABSTRACT

The waveform derived from averaging multiple EEG signal recordings during stimulation represents an Event-Related Potential (ERP). When sensory stimuli are employed, the resulting potentials are termed Evoked Potentials (EPs). EPs find applications across diverse domains of research and clinical settings, serving as a valuable tool in neuroscience and medicine due to their versatility in offering objective insights into brain function. However, the conventional signal averaging method used to extract EPs has inherent limitations, such as the necessity for numerous trials to ensure reliability and maximize Signal-to-Noise Ratio (SNR). This demands additional time for data recording and processing. Moreover, the reliability of recorded responses may be compromised due to the subject's habituation to the stimulus. To address these challenges, this study aims to enhance SNR in EP extraction by employing data augmentation, thereby reducing the number of records needed for averaging. The proposed method demonstrates a notable improvement of approximately 9.77 ± 2.65 dB compared to traditional signal averaging with the same number of records. This study concludes that judicious data augmentation enables enhanced SNR estimates without the requirement for extensive new recordings.

I. INTRODUCTION

Event-Related Potential (ERP) is a physiological phenomenon that refers to the electrical and/or magnetic responses generated in the nervous system due to a sensory, cognitive or motor stimulus. These responses are recorded using electroencephalography (EEG), magnetoencephalography (MEG) or other functional neuroimaging techniques [1]. Evoked Potentials (EP) are a subset of event-related potentials that are elicited by a specific sensory event, such as acoustic, visual or somatosensory stimuli [1]-[4]. Abnormalities have been found in the components that make up evoked responses in neurological conditions such as dementia, Parkinson's disease, multiple sclerosis, traumatic brain injury, stroke, obsessive-compulsive disorder, attention deficit hyperactivity disorder, and others. These findings are based on clinical research. It is important to note that this information is objective and does not include any subjective evaluations. Evoked Potentials have low amplitudes, ranging from 0.1 µV to 10 µV, and are embedded in the background EEG activity, which has amplitudes ranging from 10 µV to 100 µV, making it the main source of noise. Additionally, recording EEG signals can be accompanied by various artifacts and interferences that affect the accurate estimation of the potential waveform. The presence of noises and interferences can result in a very low Signal to Noise Ratio (SNR), which can be as low as -30 dB for certain types of evoked potentials, making waveform estimation challenging. To obtain the Evoked Potential signal embedded in the background noise [5]–[13] and increase the SNR, the traditional technique used is Signal averaging. This technique involves averaging the matrix formed with the individual records of the responses to each stimulus, also known as Ensemble Average.

I.1 ENSEMBLE AVERAGE AND SIGNAL TO NOISE RATIO

The Ensemble Average using the arithmetic mean transforms the observed signal into a set of *M* epochs (ensemble matrix). Each epoch contains the response to the stimulus plus noise, as shown in Figure 1. The upper part of Figure 1 corresponds to the complete signal, which is a simulated, non-realistic signal with a low noise level, used to illustrate the classical procedure.

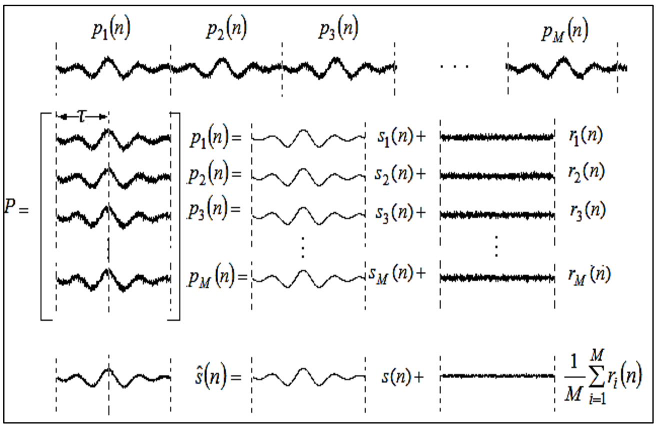

Figure 1: Ensemble average
Source: authors, (2024).

Each epoch, p_i, is considered to be the sum of the deterministic component of the signal or evoked response, *s*, which is assumed to be invariant in each epoch, and a random noise, r_i, which is asynchronous with the stimulus, as described in equation 1.

$$p_i(n) = s_i(n) + r_i(n) \tag{1}$$

Each epoch $p_i(n)$ consists of *N* samples (see equation 2) [14]-[21].

$$p_i(n), \quad i = 1, \cdots, M; \quad n = 0, \cdots, N-1 \tag{2}$$

Equation 3 shows that the estimated signal $\hat{s}$ can be modelled as the sum of the deterministic component and the average noise of all segments.

$$\begin{aligned} \hat{s}(n) &= \frac{1}{M}\sum_{i=1}^{M} p_i(n) = \frac{1}{M}\sum_{i=1}^{M}\left(s_i(n) + r_i(n)\right) \\ &= \frac{1}{M}\sum_{i=1}^{M} s_i(n) + \frac{1}{M}\sum_{i=1}^{M} r_i(n) = s(n) + \frac{1}{M}\sum_{i=1}^{M} r_i(n) \end{aligned} \tag{3}$$

To improve the signal-to-noise ratio (SNR) by a factor of $\sqrt{M}$, as expressed in equation 4 [22], it is necessary to obtain adequate noise reduction and increase the number of epochs forming the ensemble matrix [23]. Equation 4 refers to the initial signal-to-noise ratio as SNR_i and SNR_e to that estimated after averaging.

$$SNR_e = SNR_i \cdot \sqrt{M} \tag{4}$$

The SNR value of the evoked potential can be estimated from equation 5.

$$SNR = \frac{\sum_{j=1}^{N} s^2[j]}{\sum_{j=1}^{N} \theta^2[j]} \tag{5}$$

where *N* is the total number of samples of the segment to be evaluated, θ is the remaining noise in the signal (signal obtained after attenuation of the noise minus the ideal signal). The subscript *j* refers to the *j*-th sample of the parameter in question and *s* to the pure ideal signal.

Most EPs have a much smaller amplitude than the EEG signal in which they are embedded, in the range of up to -30 dB [6]. To achieve adequate noise reduction using ensemble averaging in this case, approximately 1,000 epochs are required to equate the signal power to the noise power, a ratio of 1:1, the minimum required to indicate a response [24]. The need for such a large number of epochs is one of the main limitations of averaging as a noise reduction technique in the context of PE waveform estimation [7], [25]-[28].

II DATA AUGMENTATION

Data augmentation has found its application in various classification and machine learning tasks, where data is typically scarce and difficult to obtain [29]-[33]. For electroencephalographic data, the lack of sufficient data remains a major problem. Data augmentation in EEG signals is performed by temporal, spatial/rotational transformations of the original data. Based on the characteristics of evoked potentials, if it is considered that these are signals that contain a certain periodicity (quasi-periodic) and this is associated with the synchronisation they have with the stimulus that causes them, the responses to the stimuli are practically the same. The data can be augmented by temporal transformations of the original recordings, including variations in both the latency (jitter) and the amplitude and width of the components of the evoked responses. It is then possible to obtain new recordings from the originals, including these variations. In this case, data augmentation is used to increase the size of the ensemble matrix and replace the need to record new epochs.

III. MATERIALS AND METHODS

III.1 DATA AUGMENTATION

To reduce the number of records to be averaged, data augmentation involves including temporary transformations in the initial records. These transformations advance or delay the records to simulate real-world variability in latencies. The original records and those obtained from the transformations are combined to create a new set matrix, called the augmented set matrix *A*. Equation 6 visualises an example of an augmented matrix with a shift of one sample to the right and one to the left. A small space has been left between the original matrix and its two versions to facilitate understanding of the equation. The augmented matrix will have (2d+1)M epochs and the same number of samples as the original matrix P, where d is the number of displacements in samples made from the original version. If increasing the size of the ensemble matrix is known to improve noise reduction, one might assume that a large number of shifted versions of the original matrix would solve the problem. However, this is not the case.

$$A = \begin{bmatrix} p_{1,1} & p_{1,2} & p_{1,3} & \cdots & p_{1,N-1} & p_{1,N} \\ p_{2,1} & p_{2,2} & p_{2,3} & \cdots & p_{2,N-1} & p_{2,N} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{M,1} & p_{M,2} & p_{M,3} & \cdots & p_{M,N-1} & p_{M,N} \\ & & & & & \\ p_{1,2} & p_{1,3} & p_{1,4} & \cdots, & p_{1,N} & p_{1,N} \\ p_{2,2} & p_{2,3} & p_{2,4} & \cdots & p_{2,N} & p_{2,N} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{M,2} & p_{M,3} & p_{M,4} & \cdots & p_{M,N} & p_{M,N} \\ & & & & & \\ p_{1,1} & p_{1,1} & p_{1,2} & \cdots & p_{1,N-2} & p_{1,N-1} \\ p_{2,1} & p_{2,1} & p_{2,2} & \cdots & p_{2,N-2} & p_{2,N-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{M,1} & p_{M,1} & p_{M,2} & \cdots & p_{M,N-2} & p_{M,N-1} \end{bmatrix} \quad (6)$$

Averaging the new matrix A, shown in equation 6, is equivalent to combining a moving average filter with the ensemble average, where the signal estimated from the augmented matrix A is estimated according to equation 7.

$$\hat{s}[n] = \frac{1}{M}\sum_{i=1}^{M}\frac{x_{i,n-1} + x_{i,n} + x_{i,n+1}}{3} \quad (7)$$

The cut-off frequency of a moving average filter depends on the number of samples included in the averaging window, in the case of equation 6, d=1, the window size is 3. As the size of the averaging window increases, the cut-off frequency of the moving average filter decreases, which could remove important components of the signal of interest. The maximum value of the displacements, d, in samples that can be used depends on the maximum frequency components of the signal of interest (fm) and the sampling frequency (fs) at which the signal is obtained, as described in equation 8. The criterion used to determine the maximum frequency of the signal is related to the characteristics of the bandpass filters in the signal acquisition stage.

$$d < \frac{1}{2}\left(\frac{fs}{fm} - 1\right) \quad (8)$$

Figures 2, 3 and 4 correspond to the frequency responses of the Moving Average filters at sampling rates of 13,300 samples/s, 48,000 samples/s and 500 samples/s, respectively, and for maximum frequencies of 2,000 Hz, 3,000 Hz and 30 Hz. 3 000 Hz and 30 Hz respectively. These sample rate and peak rate values are taken from real database examples of AERs. In these figures you can see how a higher value of d would reduce the width of the central lobe of the filter, thereby eliminating important components of the signal of interest. Using this criterion, the maximum value of d would be approximately 3 samples for the first sampling frequency, 7 samples for the second sampling frequency and 7 samples for the third sampling frequency.

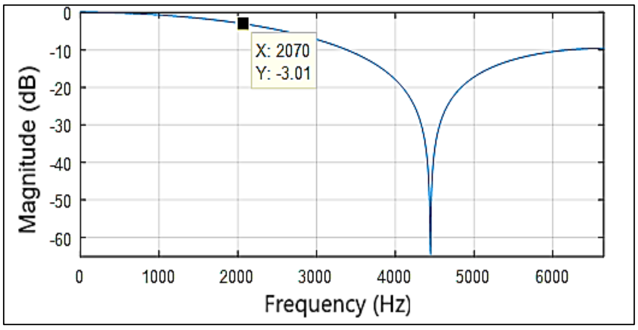

Figure 2: Frequency response of a 3-sample moving average filter at fs=13.3 kHz.
Source: authors, (2024).

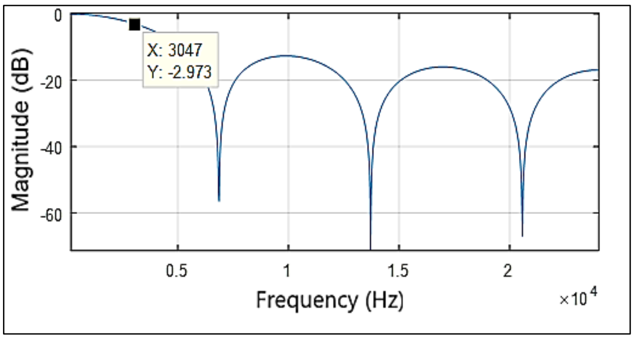

Figure 3: Frequency response of a 7-sample moving average filter at fs=48 kHz.
Source: authors, (2024).

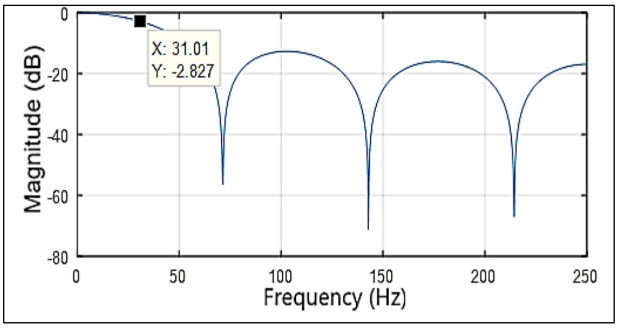

Figure 4: Frequency response of a 7-sample moving average filter at fs=500 Hz.
Source: authors, (2024).

Before averaging the augmented matrix, any elements of $A_{m,n}$ that are considered outliers are discarded. Values in each column that differ from the column median of the original matrix by more than ± 3 standard deviations from the median of the standard deviations of the estimated noise (Equation 9) in the ensemble matrix are considered outliers. The median noise deviation is estimated from equation 10.

$$q = [\tilde{p}(n) - 3\cdot\sigma_{r_{mp-mediana}},\quad \tilde{p}(n) + 3\cdot\sigma_{r_{mp-mediana}}] \quad (9)$$

$$\sigma_{r_{mp-mediana}} = \sqrt{\frac{mediana\quad \{\sigma_{up}^{2}(1, 2, \cdots, N)\}}{M}} \quad (10)$$

Then, the signal can be estimated from equation 11.

$$\hat{s}_{MRA} = \frac{\sum_{i=1}^{(2d+1)M} w_{i,n} A_{i,n}}{\sum_{i=1}^{(2d+1)M} w_{i,n}}, \quad (11)$$

where $w_{i,n}$ is the element of the *i*-th row of the *n*-th column of the weight matrix with the same size as *A*, and *d* is the number of displaced samples taken into account. The elements of the weight matrix *w* can be 0 or 1, 0 in the case that the sample is considered an outlier and 1 otherwise.

III.2 EXPERIMENT DESCRIPTION

To analyse the behaviour of the data augmentation and its influence on the waveform estimation from a smaller number of epochs, a total of 100 data sets of 2000 epochs each were generated using the simulator described in [34]. The sampling frequency of the simulated signals is 48,000 samples/s and the maximum frequency component of the signal is 3000 Hz. From each data set, 512 epochs were randomly selected 100 times to form a Monte Carlo experiment of 100 runs. In this case, the expected waveform is known a priori, so the SNR value was estimated in each run, as expressed in equation 5.

IV. RESULTS AND DISCUSSIONS

Figure 5 shows the average SNR values and their dispersion obtained for different values of d. As can be seen in Figure 5, there is a tendency for the SNR value to increase as d increases until it reaches 8, and then to decrease. This result is related to what was explained in the first part of section 2.1, d is related to the maximum frequency components of the signal and the sampling frequency, for values greater than d ≈ 7, it is not guaranteed that the shape of the signal is preserved. The initial SNR value obtained on the simulated data sets was approximately -26.0398 ± 1.16 dB. The average SNR value obtained using the classical Ensemble Average in the experiment was 0.1992 ± 1.0917 dB.

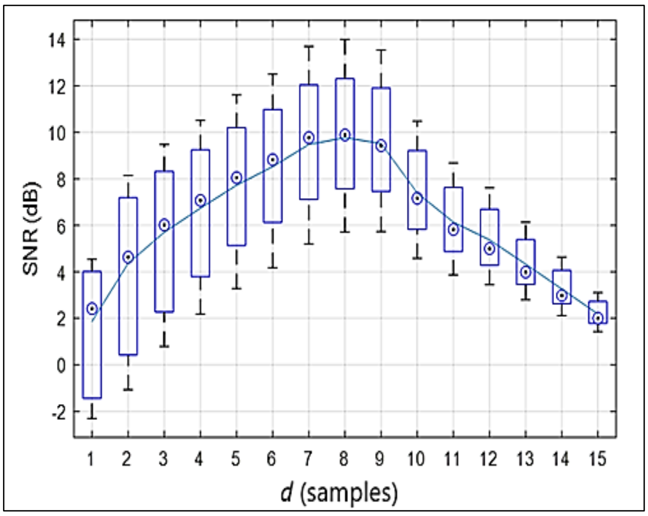

Figure 5: Signal-to-noise ratio obtained using data augmentation with different values of *d*.
Source: authors, (2024).

Figure 6 shows the result of the standard deviation of the average residual noise obtained for different values of *d*.

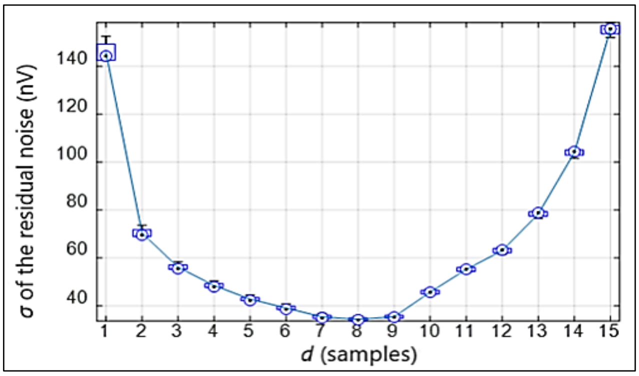

Figure 6: Standard deviation of the residual noise using data augmentation with different values of *d*.
Source: authors, (2024).

Currently, according to the protocols for PEATC [24], it is recommended that the residual noise level be equal to or less than 80 nV before concluding that there is no response. Figure 6 shows that this requirement is met for values of $d \geq 2$. A value of d = 4 results in an expanded matrix of 4608 epochs, a matrix size consistent with real-world studies. When Ensemble Average was applied to a matrix of 512 epochs, a residual noise standard deviation of 243 nV was obtained.

V. CONCLUSIONS

Data augmentation based on the characteristics of the signal can be a proposal to improve the SNR in the estimation of evoked responses with a smaller number of epochs. The results obtained in the experimental phase correspond to the theoretical ones proposed in the first part of the methodological section. The SNR values obtained in the experiment with data augmentation are about 9 dB better than those obtained with the traditional Ensemble Average method for the same number of epochs. With a value of d equal to 4, in the case of the proposed experiment, with 512 initial recordings, a number of recordings of 4608 epochs can be obtained, which is similar to the number of epochs required to be recorded in a real context. This value means that the time needed to estimate the waveform of an evoked potential is reduced by a factor of 9. On the basis of the results obtained, it is recommended that future work should evaluate the increase in data in terms of reduction of latency variability, a problem that may be resolved with this technique.

VI. AUTHOR'S CONTRIBUTION

Conceptualization: Idileisy Torres-Rodríguez, Alberto Taboada-Crispí.
Methodology: Idileisy Torres-Rodríguez, Beatriz Peón-Pérez, Alberto Taboada-Crispí.
Investigation: Idileisy Torres-Rodríguez, Beatriz Peón-Pérez, Diamir De Ávila Rodríguez, Samuel Cárdenas Herrera. Alberto Taboada-Crispí.
Discussion of results: Idileisy Torres-Rodríguez, Alberto Taboada-Crispí.
Writing – Original Draft: Idileisy Torres-Rodríguez.
Writing – Review and Editing: Idileisy Torres-Rodríguez, Beatriz Peón-Pérez, Diamir De Ávila Rodríguez, Samuel Cárdenas Herrera. Alberto Taboada-Crispí.
Resources: Idileisy Torres-Rodríguez.
Supervision: Alberto Taboada-Crispí.
Approval of the final text: Idileisy Torres-Rodríguez, Beatriz Peón-Pérez, Diamir De Ávila Rodríguez, Samuel Cárdenas Herrera. Alberto Taboada-Crispí.

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